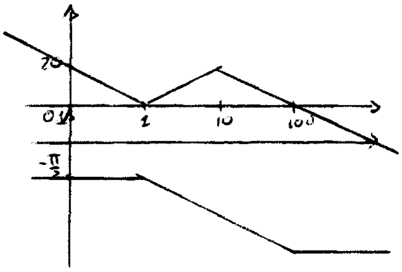
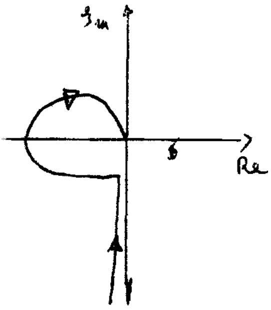
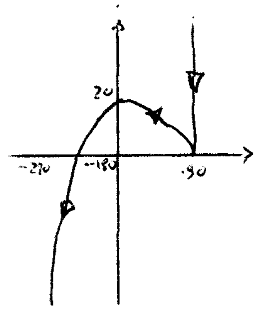
FONDAMENTI DI AUTOMATICA 11/1/10

1) $G(s) = \dfrac{(1-s)(1+s)}{s\left(1+\frac{s}{10}\right)^2}$

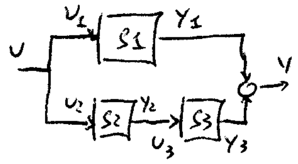






NON AS. STABILE

2)



$$S1: \begin{cases} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_1 \\ y_1 = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{cases}$$

$$S2: \begin{cases} \dot{x}_3 = -10 x_3 + u_2 \\ y_2 = 20 x_3 \end{cases}$$

$$S3: \begin{cases} \dot{x}_4 = -x_4 + u_3 \\ y_3 = x_4 \end{cases}$$

$u_1 = u$ $\quad$ $u_3 = y_2$

$u_2 = u$ $\quad$ $y = y_1 + y_3$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -10 & 0 \\ 0 & 0 & 20 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$t \leq 2$ $\quad$ $y = y_1 + y_3$

$$y_1 = |G_1(j)| \cdot 2 \cdot \sin\left(t - 0.1 + \angle G_1(j)\right) = $$
$$= 2 \sin(t - 0.1)$$

$$y_3 = |G_{23}(j)| \cdot 2 \cdot \sin\left(t - 0.1 + \angle G_{23}(j)\right) =$$
$$= 1.407 \cdot 2 \cdot \sin(t - 0.1 - 0.88)$$

$t > 2$ $\quad$ $y = y_1 + y_3$ $\quad$ y_1 è la stessa di prima

y_3 evoluzione libera di S3 a partire da $x_4(2) = y_3(2) = 2.4$

$$y_3 = 2.4\, e^{-(t-2)}\, 1(t-2)$$

3) La $F(s)$ è quella analizzata nell'esercizio 1.
Completando il diagramma polare si ha $N = -2$ e quindi non è asintoticamente stabile

4) $G(z) = \dfrac{z-1}{2\left(z+\frac{\alpha}{2}\right)}$ $\quad$ $\left|\frac{\alpha}{2}\right| < 1$ $\quad$ $|\alpha| < 2$

$$Y(z) = \frac{z-1}{2\left(z+\frac{\alpha}{2}\right)} \cdot \frac{z}{z-1} = \frac{1}{2} \frac{z}{z+\frac{\alpha}{2}}$$

$$y(k) = \frac{1}{2}\left(-\frac{\alpha}{2}\right)^k \cdot 1(k)$$